

Calculating Reliability using FIT & MTTF: Arrhenius HTOL Model

Scope

Establish a method for calculating the standard reliability values *Failure Rate* (λ), *Failures in Time* (FIT) and *Mean Time to Failure* (MTTF) using the Arrhenius *High Temperature Operating Life* (HTOL) model. The focus of this paper is to present the applicable equations, terms and definitions along with an example of an Excel driven reliability calculator used to perform these calculations.

Terms & Definitions

Reliability is defined as the probability that a component or system will continue to perform its intended function under stated operating conditions over a specified period of time.

The reliability level is derived by monitoring the functional stability of a number of representative subjects operating under elevated stress conditions resulting in a statistical prediction of reliability.

Two approaches to establishing a reliability level is to evaluate either the *probability of survival* or the *probability of failure*. Either method is equally effective, but the most common method is to calculate the *probability of failure* or *Rate of Failure* (λ). The values most commonly used when calculating the level of reliability are *FIT* (*Failures in Time*) and *MTTF* (*Mean Time to Failure*) or *MTBF* (*Mean Time between Failures*) depending on type of component or system being evaluated.

Equations & Calculations

- **Failure Rate (λ)** in this model is calculated by dividing the total number of failures or rejects by the cumulative time of operation. In the HTOL model, the cumulative time of operation is referred to as Equivalent Device Hours (EDH):

$$EDH = D \times H \times A_f$$

where: **D** = Number of Devices Tested
 H = Test Hours per Device
 A_f = Acceleration Factor derived from the Arrhenius equation

The *Failure Rate* (λ) *per hour* is shown by:

$$\lambda_{\text{hour}} = \frac{r}{D \times H \times A_f} = \frac{r}{EDH}$$

where: r = number of failures or rejects.

- **Acceleration Factor (A_f)** is the test time multiplier derived from the Arrhenius equation. This equation calculates the time acceleration value that results from operating a device at an elevated temperature. The test type used to achieve this is generally referred to as *High Temperature Operating Life* (HTOL) or *Burn-in*. More specific terms for these tests depend on the type of technology under test. Tests such as *High Temperature Reverse Bias* (HTRB), *DC Burn-in* and *Alternating Current Operating Life* (ACOL) are technology specific tests within the HTOL category. The Acceleration Factor (A_f) is shown by:

$$A_f = e^{\frac{E_a}{k} \left(\frac{1}{T_{\text{use}}} - \frac{1}{T_{\text{test}}} \right)}$$

where:

- E_a = Activation Energy (eV) of the failure mode
- k (Boltzmann's Constant) = 8.617×10^{-5} eV/°K
- T_{use} = Use Temperature (standardized at 55°C or 328°K)
- T_{test} = Test Temperature (HTOL temperature in °K)
- °K (degrees Kelvin) = $273 + ^\circ\text{C}$
- eV = electron volts

- **Activation energy (E_a)** is an empirical value that is the minimum energy required to initiate a specific type of failure mode that can occur within a technology type. Oxide defects, bulk silicon defects, mask defects, electro-migration and contamination are some examples of such failure modes, each with a unique associated E_a . In lieu of using empirical data for each individual failure mode, it is generally accepted that a standard single value of **$E_a = 0.7$ eV** provides a reasonably accurate average E_a value for diode type semiconductors.
- To derive a more statistically accurate calculation for *Failure Rate* (λ), the *number of rejects* (r) is replaced with the probability function using *Chi-squared* (X^2).

$$r \sim \frac{X^2(\alpha, \nu)}{2}$$

where: $X^2/2$ (Chi-squared/2) is the probability estimation for the number of failures or rejects.

α (**alpha**), *confidence level* (CL) or *probability*, is the applicable percent area under the X^2 probability distribution curve; reliability

calculations use $\alpha = 0.6$ (or 60%). Note: The *total area* under the X^2 curve is always 1, so $\alpha \leq 1$ (or 100%).

ν (**nu**), *degrees of freedom* (DF), determines the shape of the X^2 curve; reliability calculations use $\nu = 2r + 2$ where r = number of failures or rejects.

The equation for *Failure Rate* (λ) *per hour* then becomes:

$$\lambda_{hour} = \frac{X^2(\alpha, \nu)}{2 \times D \times H \times A_f} = \frac{X^2(\alpha, \nu)}{2 \times EDH}$$

- **FIT (Failures in Time)** is a standard industry value defined as the *Failure Rate* (λ) *per billion hours*.

$$FIT = \lambda_{FIT} = \lambda_{hours} \times 10^9$$

- **MTTF (Mean Time to Failure or θ)** is another standard industry value which provides the average time to failure of *Non-repairable Items* such as light bulbs and diodes or unserviceable systems such as satellites or other unmanned space craft. For items with long life expectancies, it is often a more useful to report MTTF in *years* rather than *hours*.

$$MTTF_{hours} = 1/\lambda_{hours} \quad \text{or} \quad MTTF_{years} = 1/(\lambda_{hours} \times 24 \times 365)$$

- **MTBF (Mean Time between Failures)** is used to describe *Repairable Items* such as compressors and aircraft. MTBF uses MTTF as one factor and *Mean Time to Repair* (MTTR) as the other to capture the complete break-down and repair cycle. The primary purpose of MTBF is to identify appropriate preventive maintenance schedules to avoid, perhaps indefinitely, catastrophic failures due to predictable piece part *wear-out*. *As a rule of thumb, component reliability centers around MTTF since most components cannot be repaired.* MTBF is shown by:

$$MTBF = MTTF + MTTR$$

where: **MTTR** is the Mean Time to Repair.

Reliability Calculation Tools

Over the years, there have been several methods used to derive the value for X^2 (Chi-squared); everything from the use of *probability tables* to the application of

approximation equations. More detail on this subject is discussed in “Calculating Chi-squared (X^2) for Reliability Equations” by this author. The following is an excerpt.

In MS Excel 2003 and later versions, the CHIINV function calculates X^2 from the input data *probability* and *deg_freedom* rendering accurate values throughout the entire use range up to 4-decimal places. This method is currently used as a standard in the statistics community for calculating values for the X^2 distribution. In Excel, the formula is expressed as:

$$=CHIINV(probability,deg_freedom)$$

In determining ν (**nu**), degrees of freedom or *deg_freedom*, we use the equation described previously where $r = \text{number of failures or rejects}$:

$$\nu = 2(r + 1) = 2r + 2$$

Since reliability calculations require the *left* portion of area under the X^2 distribution curve and CHIINV happens to calculate the *right* side, we must use the equation below to determine the correct value for *probability*:

$$CHIINV(probability) = 1 - \alpha$$

when using decimal values for α , or

$$CHIINV(probability) = 1 - \frac{\alpha}{100}$$

when using percentage values for α .

Example: for $\alpha = .60$ or 60%, $CHIINV(probability) = .40$ or 40% respectively.

Figure 1 shows an example of an Excel driven reliability calculator which greatly simplifies the calculation activity.



Reliability Calculator

MicroNote™ 1004

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Reliability Calculator using Arrhenius HTOL Model

INPUT DATA: Do not use screening data for reliability calculations.

Number of Failures (r)	Number of Devices Tested (D)	Test Hours per Device (H)
0	220	1000
1	450	380

Test Temperature (°C)	Use Temperature (°C)	Established Activation Energy (Ea)
125	55	0.7

Chi-Square (X²) @ 60% Confidence Level (α)	Acceleration Factor (Af)	Total Number of Failures (r)	Total Number of Device Hours (DH)	Equivalent Device Hours (EDH)
4.045	77.94	1	391,000	30,474,923

Failure Rate (λ _{hour})	6.64E-08
FIT : Failures in Time (λ per billion hours)	66

MTTF _{hours} : Mean Time to Failure (θ in hours)	15,069,338
MTTF _{years} : Mean Time to Failure (θ in years)	1,720

- Indicates Data Entry Point
- Indicates Variable Value

Parameters & Constants

r = number of failures or rejects
 D = Number of Devices Tested
 H = Test Hours per Device
 T (Calculation Temperature in Kelvin) = Celsius + 273°
 T_{test} = Test Temperature (HTRB or other burn-in temperature)
 T_{use} = Use Temperature (standardized at 55°C or 328°K)
 E_a = Activation Energy (eV)
 X²/2 (Chi-Squared/2) is the probability estimation for number of failures at α and ν
 α (alpha) = Confidence Level for X² distribution; reliability calculations use α = 60% or .60
 ν (nu) = Degrees of Freedom for X² distribution; reliability calculations use ν = 2r + 2
 A_f = Acceleration Factor from the Arrhenius equation
 k (Boltzmann's Constant) = 8.617 x 10e-5 eV/°K
 DH (Device Hours) = D x H
 EDH (Equivalent Device Hours) = D x H x A_f
 λ_{hour} = Failure Rate per hour
 FIT (λ_{FIT}) = Failures in Time = Failure Rate per billion hours
 MTTF = Mean Time to Failure

Equations

$$A_f = e^{\frac{E_a}{k} \left(\frac{1}{T_{use}} - \frac{1}{T_{test}} \right)} \quad FIT = \lambda_{FIT} = \lambda_{hours} \times 10^9$$

$$\lambda_{hour} = \frac{X^2(\alpha, \nu)}{2 \times D \times H \times A_f} = \frac{X^2(\alpha, \nu)}{2 \times EDH}$$

$$MTTF_{hours} = \frac{1}{\lambda_{hours}} \quad MTTF_{years} = \frac{1}{(\lambda_{hours} \times 24 \times 365)}$$

Reference Material

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